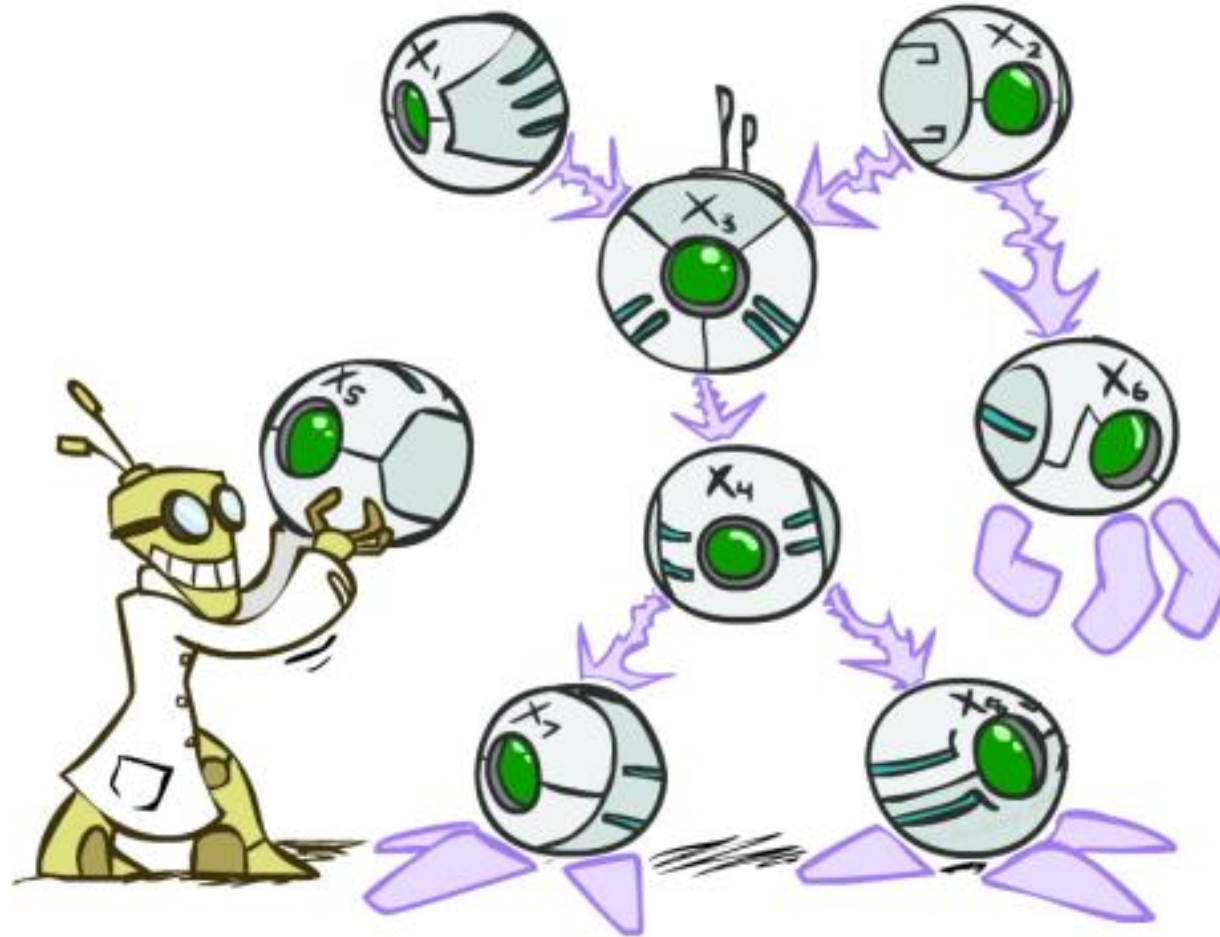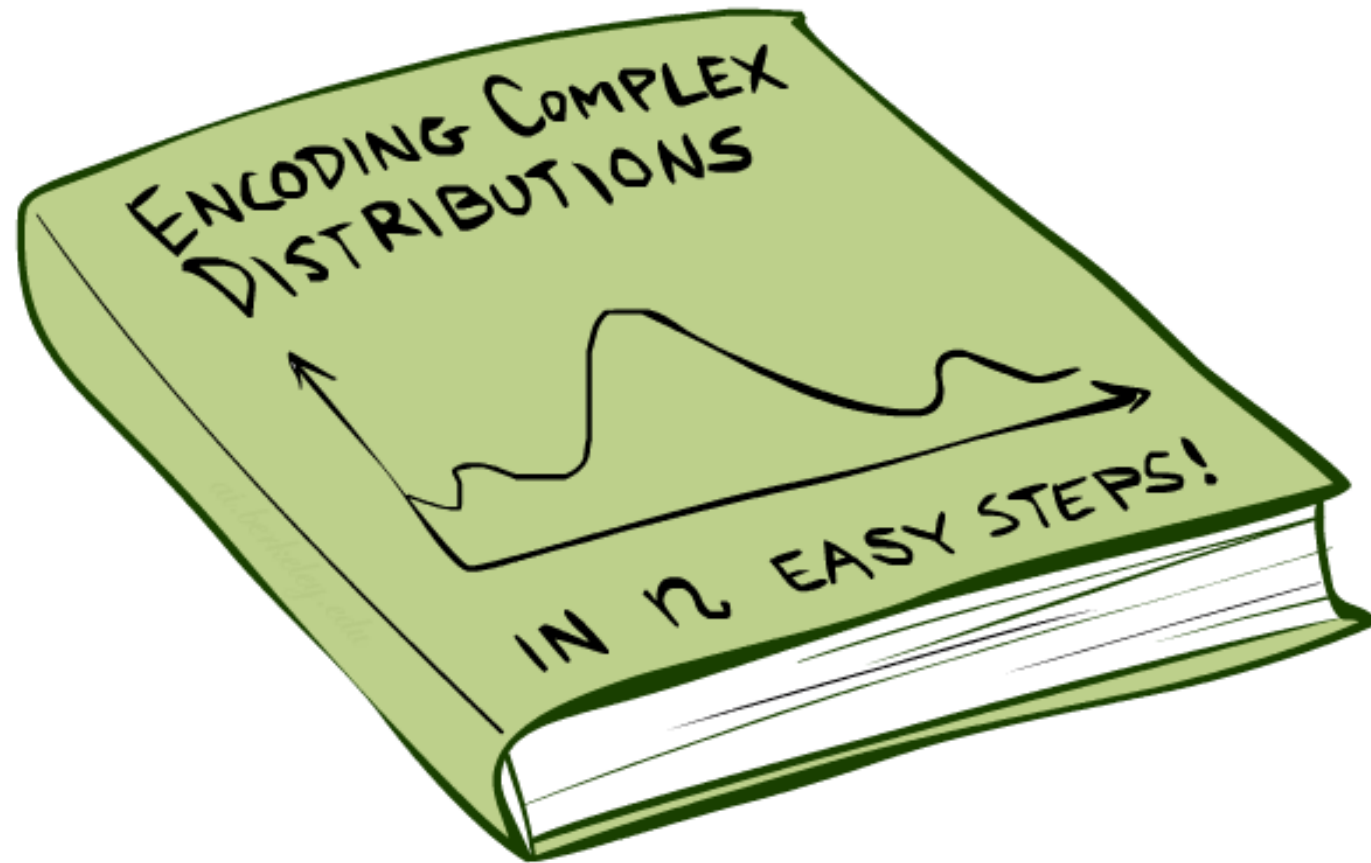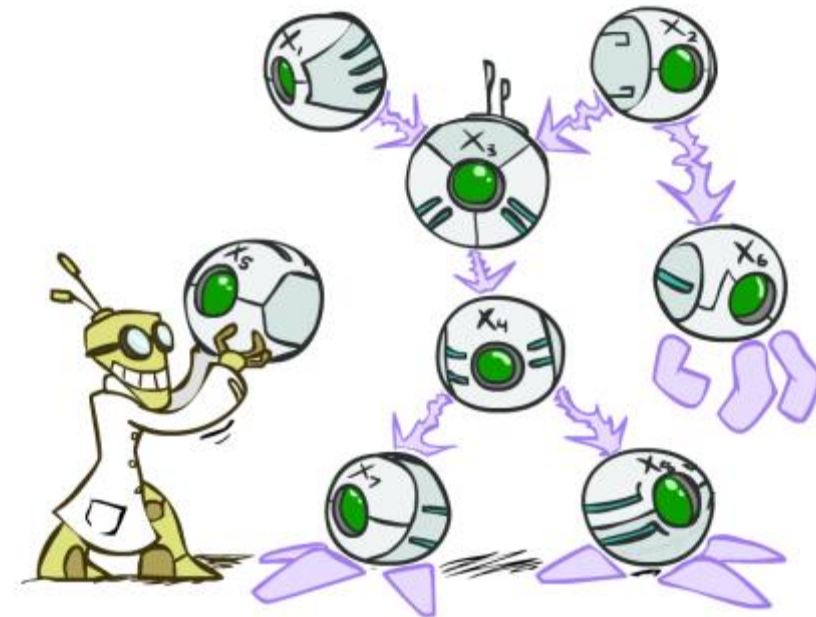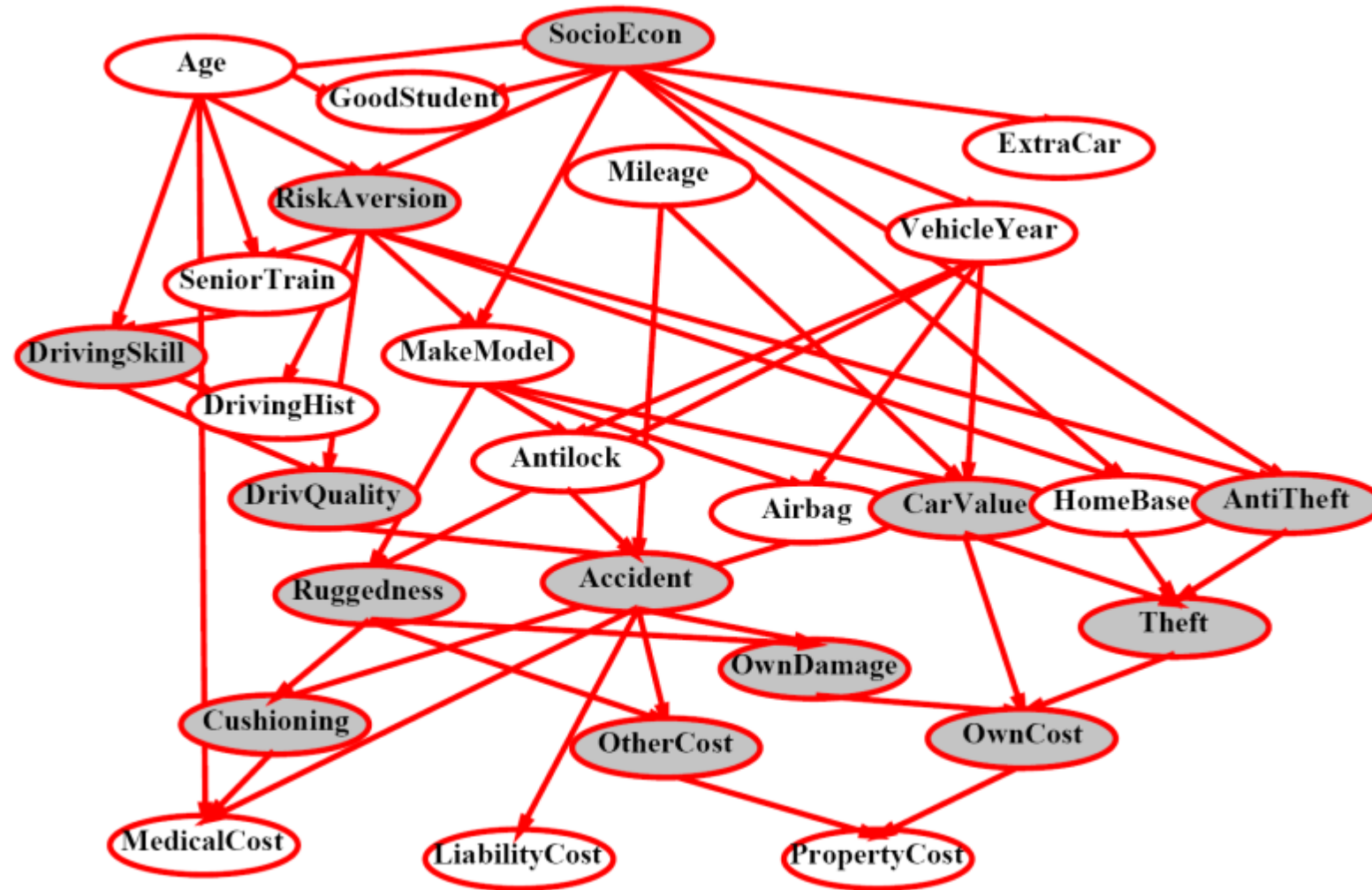# Bayes' Nets

Bayes' Nets: Big Picture

Bayes' Nets: Big Picture

- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time

- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called graphical models
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions
  - For about 10 min, we'll be vague about how these interactions are specified

Example Bayes' Net: Insurance

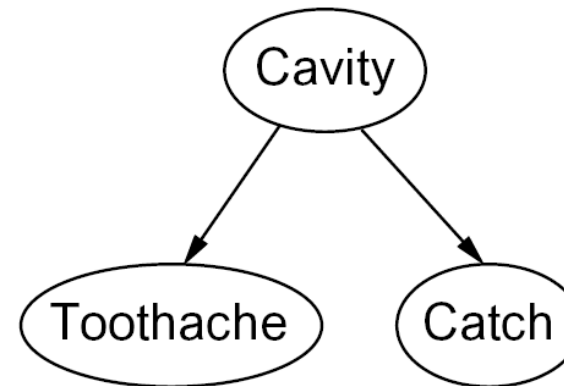# Example Bayes' Net: Car
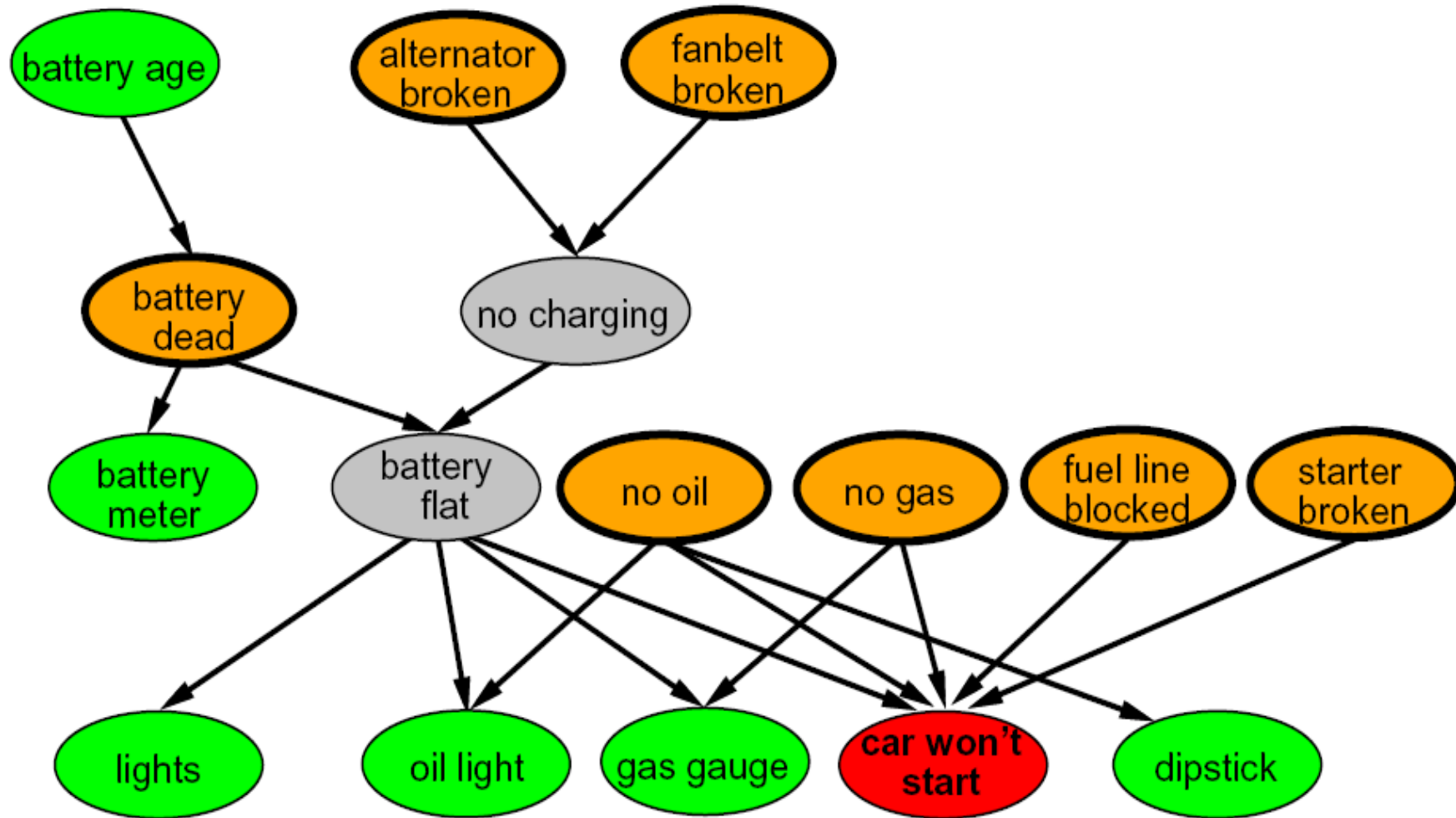
Graphical Model Notation

- **Nodes: variables (with domains)**
  - Can be assigned (observed) or unassigned (unobserved)

- **Arcs: interactions**
  - Similar to CSP constraints
  - Indicate "direct influence" between variables
  - Formally: encode conditional independence (more later)

- **For now: imagine that arrows mean direct causation (in general, they don't!)**
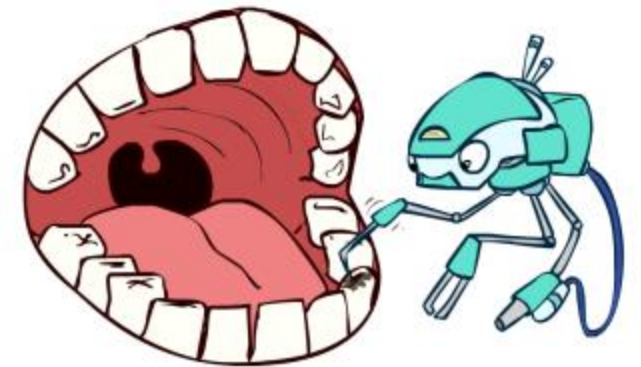
Example: Coin Flips

- N independent coin flips

$$X_1 \quad X_2 \quad \cdots \quad X_n$$
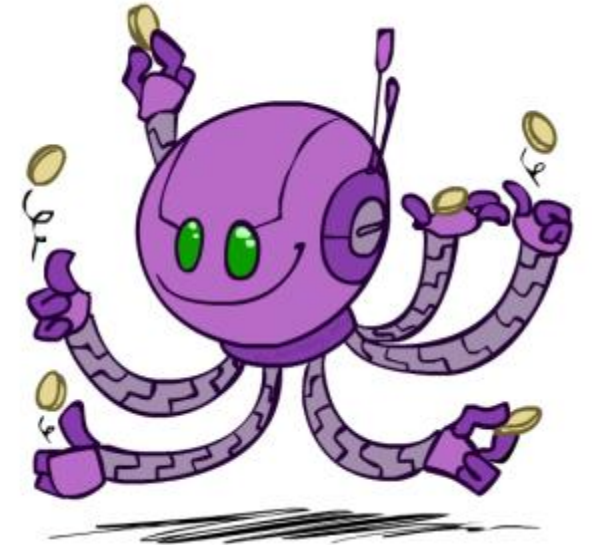
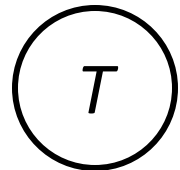- No interactions between variables: absolute independence

Example: Traffic

- Variables:
  - R: It rains
  - T: There is traffic

- Model 1: independence

R

T
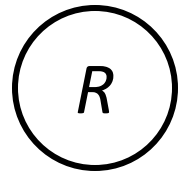
- Model 2: rain causes traffic

R

T
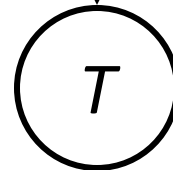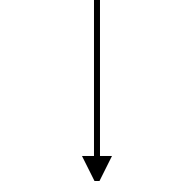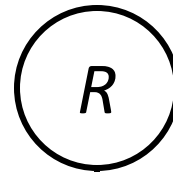
- Why is an agent using model 2 better?

Example: Traffic II

- Let's build a causal graphical model!
- Variables:
  - T: Traffic
  - R: It rains
  - L: Low pressure
  - D: Roof drips
  - B: Ballgame
  - C: Cavity
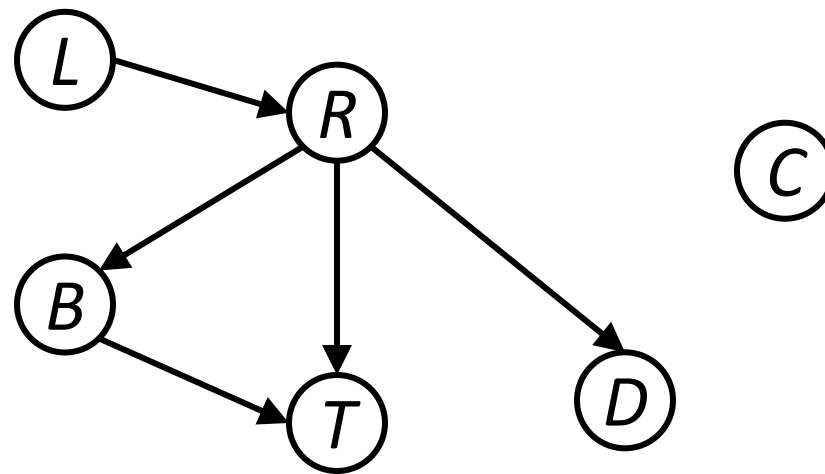
Example: Alarm Network

- Variables:
  - B: Burglary
  - A: Alarm goes off
  - M: Mary calls
  - J: John calls
  - E: Earthquake!
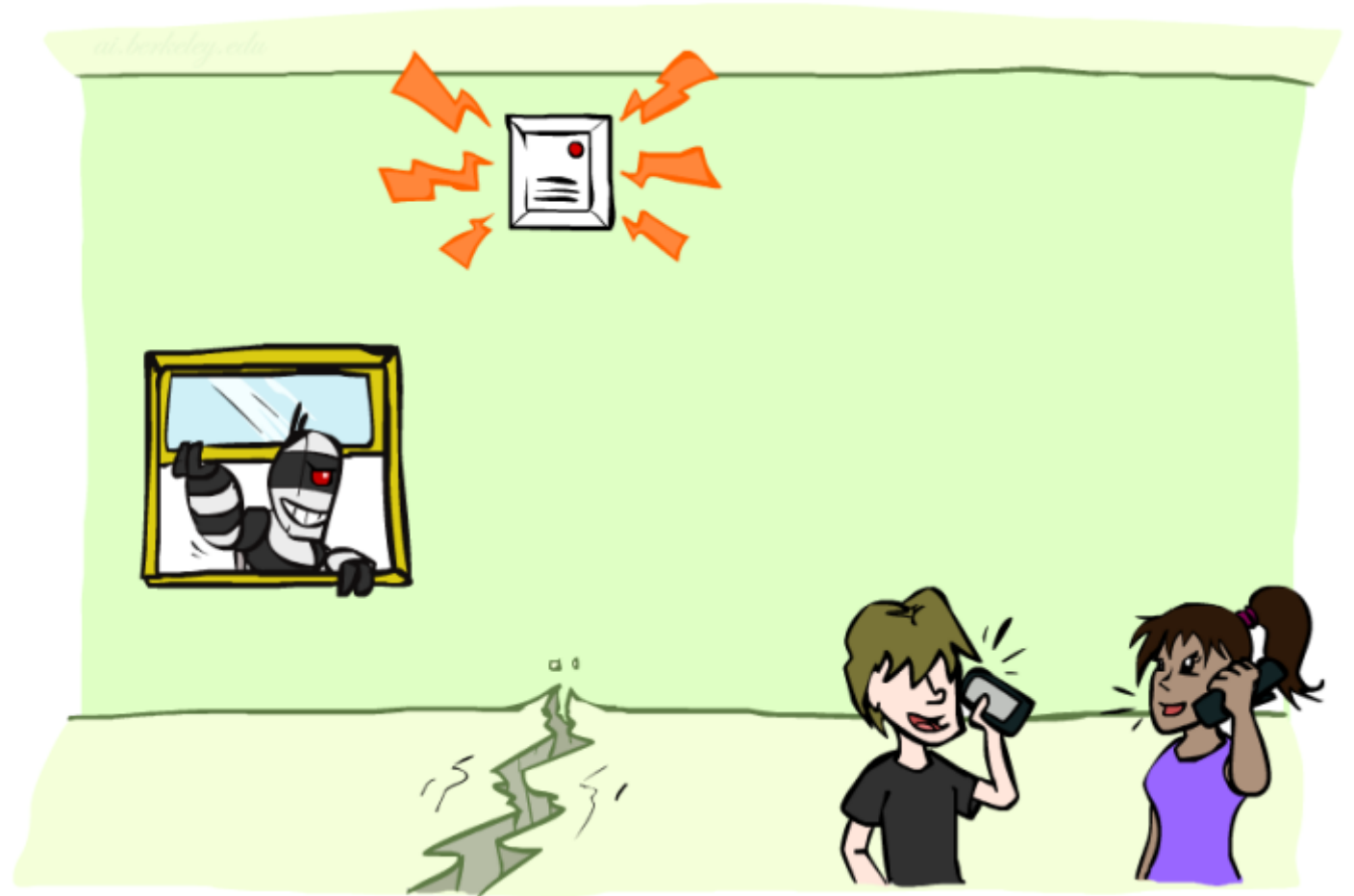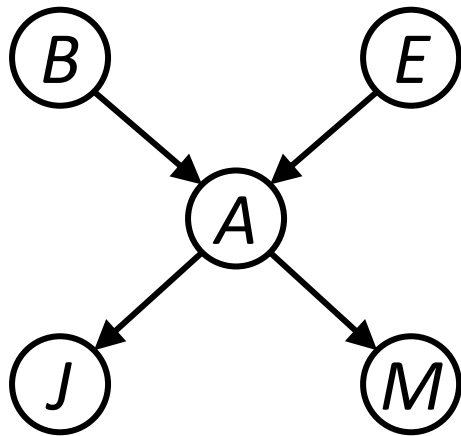
Bayes' Net Semantics

Bayes' Net Semantics

- A set of nodes, one per variable X

- A directed, acyclic graph

- A conditional distribution for each node

  - A collection of distributions over X, one for each combination of parents' values

  $$P(X|a_1 \ldots a_n)$$

  - CPT: conditional probability table

  - Description of a noisy "causal" process

$$P(X|A_1 \ldots A_n)$$
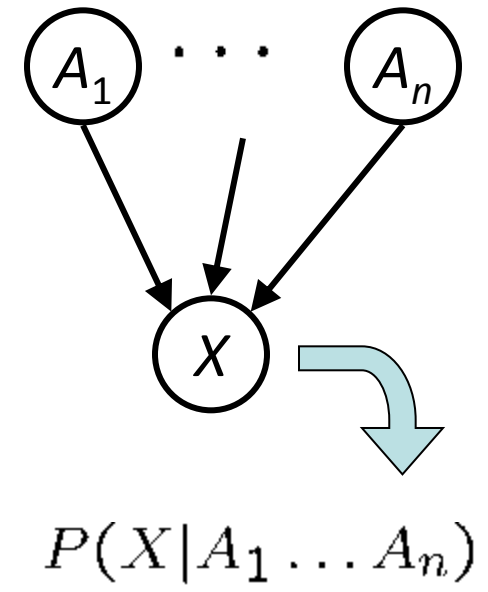
A Bayes net = Topology (graph) + Local Conditional Probabilities

Probabilities in BNs

- Bayes' nets **implicitly** encode joint distributions

  - As a product of local conditional distributions

  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

  $$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

  - Example:
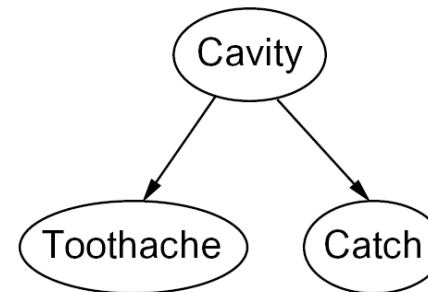
P(+cavity, +catch, -toothache) = P(+cavity) P(+catch | +cavity) P(-toothache | +cavity)

Probabilities in BNs

- Why are we guaranteed that setting

$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions):
$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | x_1 \ldots x_{i-1})$$

- <u>Assume</u> conditional independences:
$$P(x_i | x_1, \ldots x_{i-1}) = P(x_i | parents(X_i))$$

→ Consequence:
$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

- Not every BN can represent every joint distribution
  - The topology enforces certain conditional independencies

Example: Coin Flips

$P(X_1)$

h	0.5
t	0.5

$P(X_2)$

h	0.5
t	0.5

. . .

$P(X_n)$

h	0.5
t	0.5
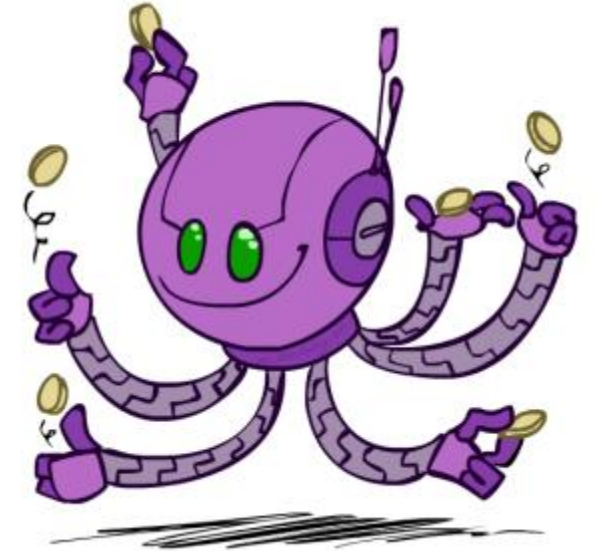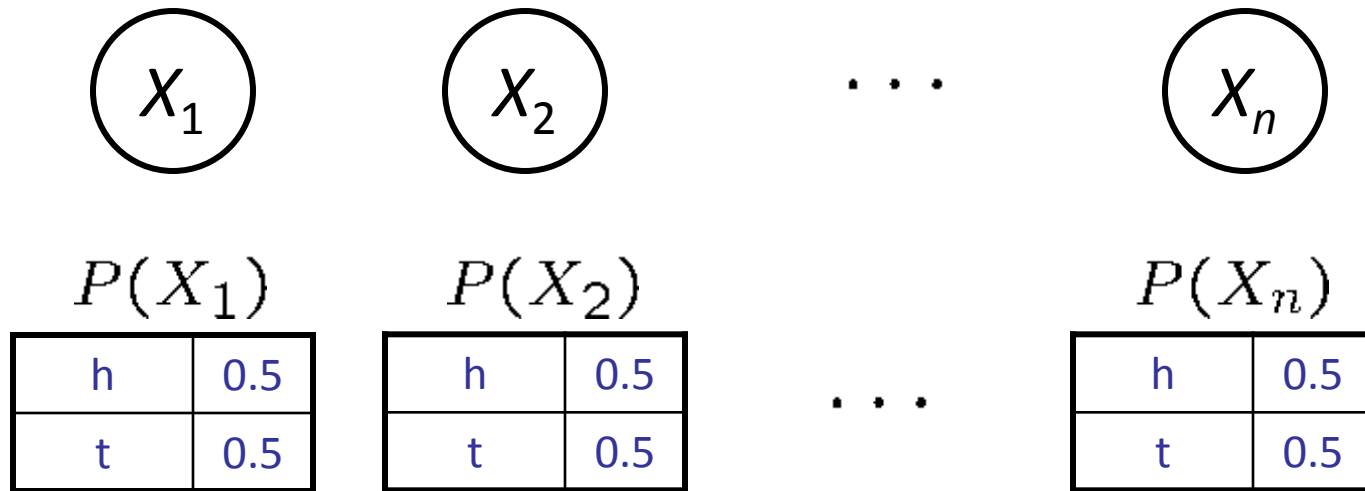
$P(h, h, t, h)$ $= P(X_1) P(X_2) P(X_3) P(X_4) = P(h) P(h) P(t) P(h) = 0.5^4$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.

Example: Traffic

$P(R)$

+r	1/4
-r	3/4

$P(T|R)$
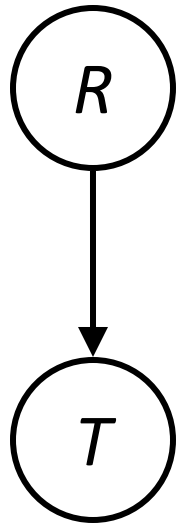
+r	+t	3/4
	-t	1/4

-r	+t	1/2
	-t	1/2

P(+r, -t) = P(+r) P(-t|+r) = 0.25 (0.25)

Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999

B: Burglary

E: Earthquake

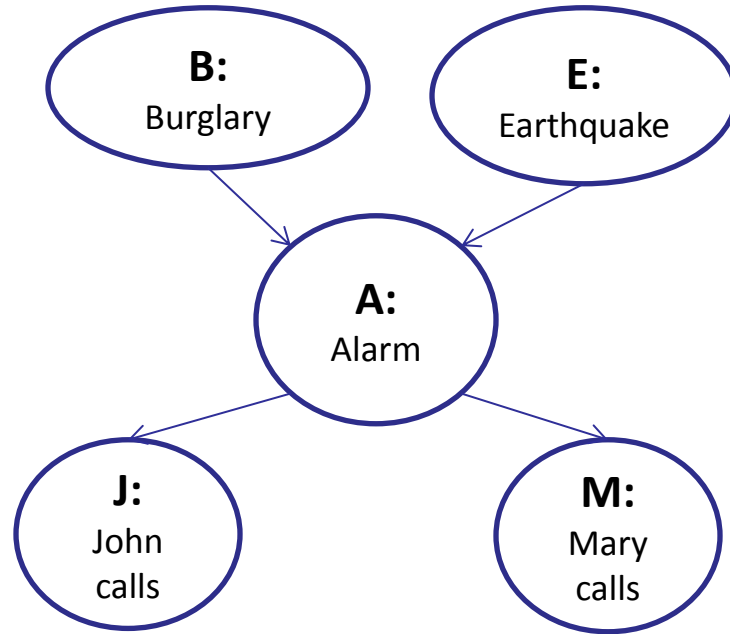| E	P(E)
+e	0.002
-e	0.998

A: Alarm

J: John calls

M: Mary calls

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

$$P(+b, -e, +a, +j\ +m) = P(+b)\ P(-e)\ P(+a|+b,-e)\ P(+j|+a)\ P(+m|+a)$$

Example: Traffic

- Causal direction

$P(R)$

+r	1/4
-r	3/4

$P(T|R)$
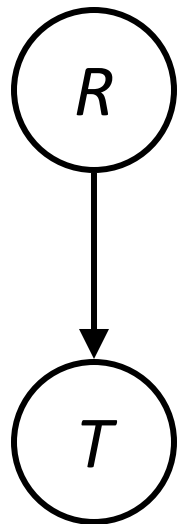
+r	+t	3/4
	-t	1/4
-r	+t	1/2
	-t	1/2

$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

Example: Reverse Traffic

- Reverse causality?
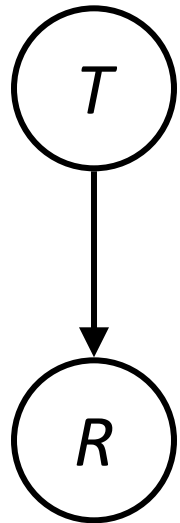
$P(T)$

+t	9/16
-t	7/16

$P(R|T)$

+t	+r	1/3
	-r	2/3

| -t	+r	1/7
	-r	6/7

$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

Causality?

- When Bayes' nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts

- BNs need not actually be causal
  - Sometimes no causal net exists over the domain (especially if variables are missing)
  - E.g. consider the variables *Traffic* and *Drips*
  - End up with arrows that reflect correlation, not causation

- What do the arrows really mean?
  - Topology may happen to encode causal structure
  - Topology really encodes conditional independence
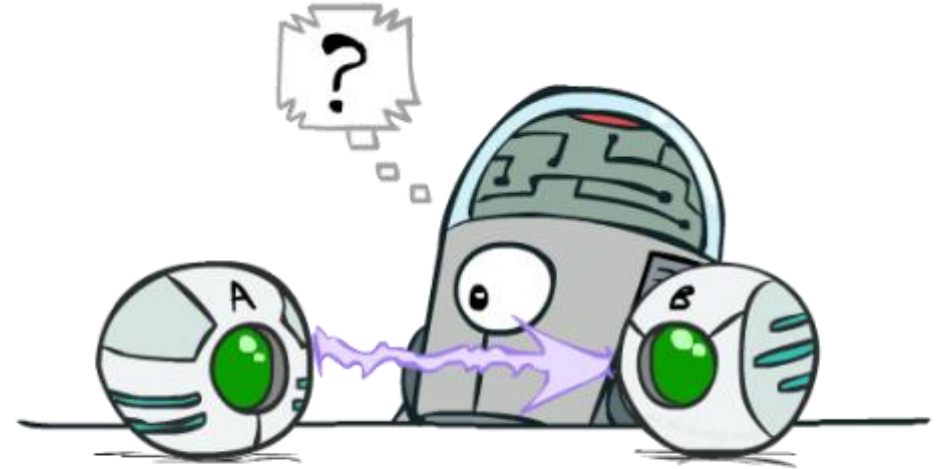
$$P(x_i | x_1, \ldots x_{i-1}) = P(x_i | parents(X_i))$$

Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution

- Next: how to answer queries about that distribution
  - Today:
    - First assembled BNs using an intuitive notion of conditional independence as causality
    - Then saw that key property is conditional independence
  - Main goal: answer queries about conditional independence and influence

- After that: how to answer numerical queries (inference)